BMath-II-Group Theory-Midsem Exam

INSTRUCTIONS: Total time 2 hours. Solve problems for a maximum score of 30 marks. Please use notations and terminology as developed in the course, use results done in the class without proving them. If you use a problem from some assignment/quiz/homework/book, please provide its solution.

1. Let $n \geq 2$ be an integer. Let $\mathbb{C}^{\times}$ denote the multiplicative group of all nonzero complex numbers. Determine all group homomorphisms $S_n \longrightarrow \mathbb{C}^{\times}$, where S_n is the symmetric group. (8)
2. Let G be a finite cyclic group of order n. Prove that for any divisor d of n, G has a unique subgroup of order d. (8)
3. Show that for any integer $n \geq 1$, $n = \sum_{d|n} \phi(n)$, where ϕ is the Euler's phi-function. (8)
4. Let G be a finite group having exactly two conjugacy classes. Find the order of G. (6)
5. Let G be a group of order 231. Prove that the Sylow-11 subgroup is contained in the center of G. (6)
6. Find out, with justification, which of the pairs of groups is a pair of nonisomorphic groups: **(i)** $\{(\mathbb{Q}, +), (\mathbb{Q}^{\times}, \cdot)\}$ **(ii)** $\{(\mathbb{R}, +), (\mathbb{R}^{>0}, \cdot)\}$, where $(\mathbb{R}^{>0}, \cdot)$ is the multiplicative group of positive real numbers. (6+6)
7. Prove that a group of order 108 or 148 cannot be simple. (8)